部分積分法

$$\{f(x)g(x)\}' = f'(x)g(x) + f(x)g'(x)$$

$$f(x)g'(x) = \{f(x)g(x)\}' - f'(x)g(x)$$

積分

$$\int f(x)g'(x)\,dx = \int \{f(x)g(x)\}'\,dx - \int f'(x)g(x)\,dx$$

$$\boxed{\int f(x)g(x)\,dx = f(x)g(x) - \int f'(x)g(x)\,dx}$$

例題4

$$\boxed{\int \boxed{x}\,\boxed{\sin 2x}\,dx}$$

$\boxed{x} \to f(x)$　　$\boxed{\sin 2x} \to g'(x)$

$$= \int \boxed{x}\,\boxed{\left(-\frac{1}{2}\cos 2x\right)'}\,dx$$

$g(x)$　　$f'(x)$

$$= -\frac{1}{2}x\cos 2x - \int \boxed{1}\cdot\left(-\frac{1}{2}\cos 2x\right)dx$$

$$= -\frac{1}{2}x\cos 2x + \frac{1}{2}\cdot\frac{1}{2}\sin 2x + C$$

$$= \frac{1}{4}\sin 2x - \frac{1}{2}\cos 2x + C$$

$$\int x\sin 2x\,dx$$

$$\left(\begin{array}{ll} f = x & f' = 1 \\ g' = \sin 2x & g = -\frac{1}{2}\cos 2x \end{array}\right)$$

①　②

$$= -\frac{1}{2}x\cos 2x - \int -\frac{1}{2}\cos 2x\,dx$$

$$\int x\sin 2x\,dx = \frac{1}{2}x^2\sin 2x - \int x^2\cos 2x\,dx$$

また部分積分!?

$$\left(\begin{array}{ll} f = \sin 2x & f' = 2\cos 2x \\ g' = x & g' = \frac{1}{2}x^2 \end{array}\right)$$

Point

部分積分は

$\int \square\, dx$ の $\square$ が部分積分にならないように、f と g' を設定する!

例題4　続き

$$\boxed{\int \frac{x}{\cos^2 x}\,dx}$$

$$\left(\begin{array}{ll} f = x & f' = 1 \\ g' = \dfrac{1}{\cos^2 x} & g = \tan x \end{array}\right)$$

$$= x\tan x - \int \tan x\,dx$$

$$= x\tan x - \int \frac{\sin x}{\cos x}\,dx$$

ここで $\int \frac{\sin x}{\cos x}\,dx$ について

$\cos x = t$ とおくと

$$-\sin x\,dx = dt$$

$$\sin x\,dx = -dt$$

$$\therefore \int \frac{\sin x}{\cos x}\,dx = \int -\frac{1}{t}\,dt$$

$$= -\log|t| + C$$

$$= -\log|\cos x| + C$$

$$\therefore (与式) = \underline{x\tan x + \log|\cos x| + C}$$

例題4 もういっちょ

$$\boxed{\int x^3 \log x \, dx}$$

$$\left(\begin{array}{ll} f = \log x & f' = \dfrac{1}{x} \\ g' = x^3 & g = \dfrac{1}{4}x^4 \end{array} \right)$$

$$= \frac{1}{4}x^4 \log x - \int \frac{1}{4}x^3 \, dx$$

$$= \underline{\frac{1}{4}x^4 \log x - \frac{1}{16}x^4 + C}$$

例題4 やれるもんなら.

$$\boxed{\int \log x \, dx}$$

$$\left(\begin{array}{ll} f = \log x & f' = \dfrac{1}{x} \\ g' = 1 & g = x \end{array} \right)$$

$$= x \log x - \int 1 \, dx$$

$$= \underline{x \log x - x + C}$$

例題5

$$\boxed{\int x^2 e^{-x}\,dx}$$

$$\begin{pmatrix} f = x^2 & f' = 2x \\ g' = e^{-x} & g = -e^{-x} \end{pmatrix}$$

$$= -x^2 e^{-x} + \int 2x e^{-x}\,dx$$

$$\begin{pmatrix} f = 2x & f' = 2 \\ g' = e^{-x} & g = -e^{-x} \end{pmatrix}$$

$$= -x^2 e^{-x} + \left\{ -2x e^{-x} + \int 2e^{-x}\,dx \right\}$$

$$= -x^2 e^{-x} - 2x e^{-x} - 2e^{-x} + C$$

$$= -(x^2 + 2x + 2)e^{-x} + C$$