

いろいろな関数の不定積分

例題 6

$$\int \frac{x^2 + 3x + 4}{x + 2} dx$$

$$(\text{与式}) = \int \left(x + 1 + \frac{2}{x + 2} \right) dx$$

X-1

$$\begin{array}{r} x + 1 \\ x + 2 \overline{) x^2 + 3x + 4} \\ \underline{\ominus x^2 + 2x} \\ x + 4 \\ \underline{\ominus x + 2} \\ 2 \end{array}$$

$$= \frac{1}{2} x^2 + x + 2 \log |x + 2| + C$$

$$\frac{(x + 2)(x + 1) + 2}{x + 2}$$

例題6 続き

$$\int \frac{dx}{x^2-4}$$

$$= \int \frac{dx}{(x-2)(x+2)}$$

xを

$$\frac{1}{(x-2)(x+2)} = \frac{1}{4} \left(\frac{1}{x-2} - \frac{1}{x+2} \right)$$

$$= \frac{1}{4} \frac{4}{(x-2)(x+2)}$$

$$= \int \frac{1}{4} \left(\frac{1}{x-2} - \frac{1}{x+2} \right) dx$$

$$= \frac{1}{4} \log \left| \frac{x-2}{x+2} \right| + C$$

$$= \frac{1}{4} (\log|x-2| - \log|x+2|) + C$$

例題 6 もじいっちょ.

$$\int \frac{x}{(x-1)(2x-1)} dx$$

部分分数分解できる!

$$\frac{x}{(x-1)(2x-1)} \text{ について}$$

$$\frac{x}{(x-1)(2x-1)} = \frac{a}{x-1} + \frac{b}{2x-1}$$

とおく.

$$\begin{aligned} \frac{a}{x-1} + \frac{b}{2x-1} &= \frac{a(2x-1)+b(x-1)}{(x-1)(2x-1)} \\ &= \frac{(2a+b)x-a-b}{(x-1)(2x-1)} \end{aligned}$$

恒等式より.

$$(2a+b)x-a-b = x$$

$$\begin{cases} 2a+b=1 \\ -a-b=0 \end{cases}$$

$$a=1$$

$$b=-1$$

$$\therefore \frac{1}{x-1} - \frac{1}{2x-1}$$

$$(\frac{5}{7} \text{式}) = \int \left(\frac{1}{x-1} - \frac{1}{2x-1} \right) dx$$

$$= \log|x-1| - \frac{1}{2} \log|2x-1| + C$$
