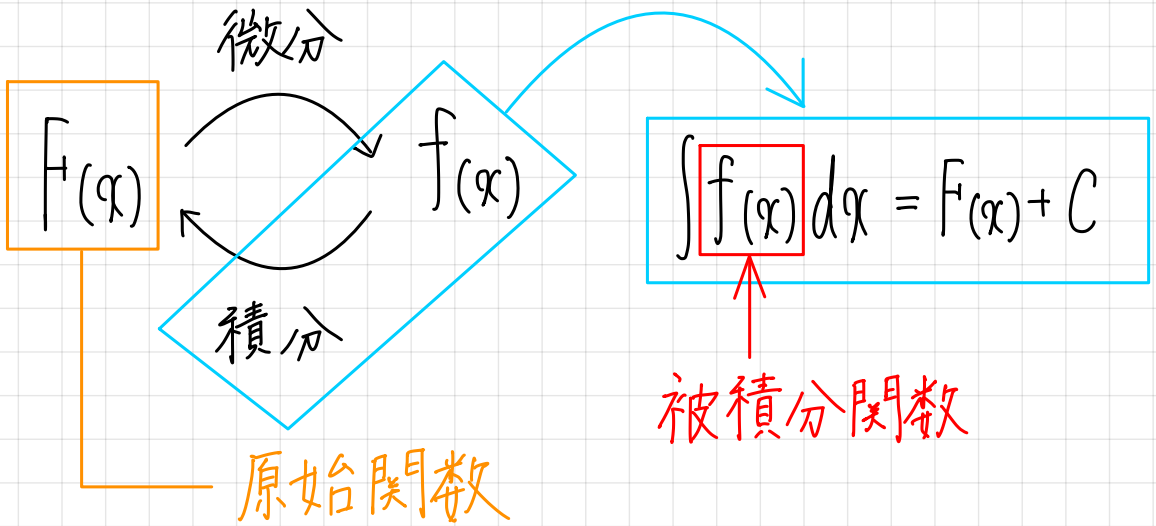
第7章 積分

不定積分

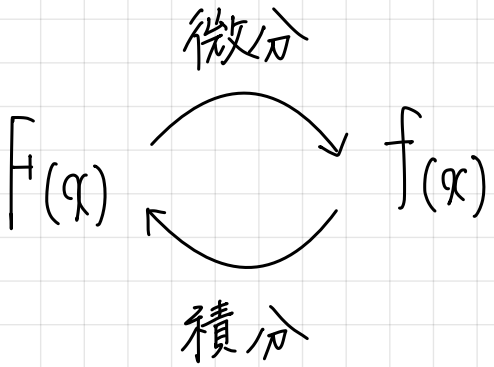


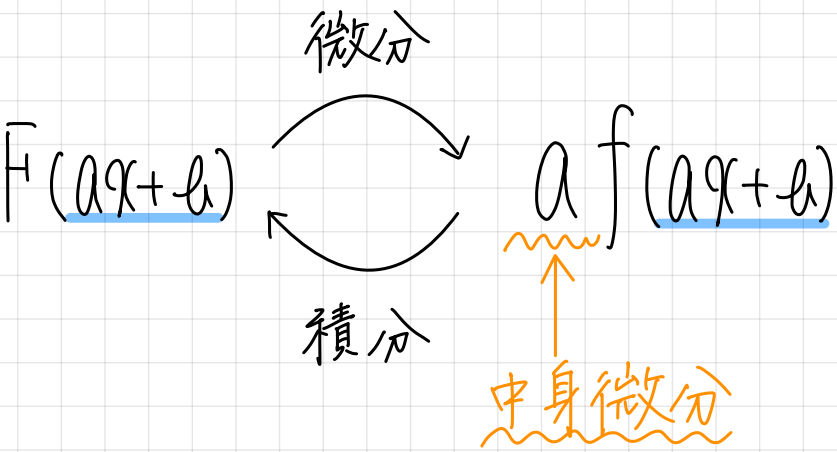
導関数の逆を利用

公式

- $\int x^{\alpha} dx = \frac{1}{\alpha+1} x^{\alpha+1} + C$
- $\int \frac{1}{x} dx = \log|x| + C$
- $\int \sin x \, dx = -\cos x + C$
- $\int \cos x \, dx = \sin x + C$
- $\int \frac{1}{\cos^2 x} dx = \tan x + C$
- $\int e^x dx = e^x + C$
- $\int a^x dx = \frac{a^x}{\log a} + C$

$f(ax+b)$の積分





$$\int f(ax+b)\,dx = \frac{1}{a}F(ax+b) + C$$

置換積分法

例題1

不定積分 $\int x\sqrt{x+1}\,dx$

$\sqrt{x+1}=t$ とおくと

$x+1=t^2$

$x=t^2-1$

$dx=2t\,dt$

$$(与式)=\int(t^2-1)t\cdot 2t\,dt$$

$$=\int(2t^4-2t^2)\,dt$$

$$=\frac{2}{5}t^5-\frac{2}{3}t^3+C$$

$$=\frac{2}{15}t^3(3t^2-5)+C$$

$$=\frac{2}{15}(3x-2)(x+1)\sqrt{x+1}+C$$

例題1 続き

$$\int x\sqrt{x+2}\,dx$$

$\sqrt{x+2}=t$ とおく

$x+2=t^2$

$x=t^2-2$

$dx=2t\,dt$

$$(与式)=\int(t^2-2)\cdot t\cdot 2t\,dt$$

$$=\int(2t^4-4t^2)\,dt$$

$$=\frac{2}{5}t^5-\frac{4}{3}t^3+C$$

$$=\frac{1}{15}t^3(6t^2-20)+C$$

$$=\frac{2}{15}t^3(3t^2-10)+C$$

$$=\frac{2}{15}(x+2)\sqrt{x+2}\,(3x+6-10)+C$$

$$=\frac{2}{15}(3x-4)(x+2)\sqrt{x+2}+C$$

例題1　もういっちょ

$$\int 3(x^3+2)x^2\,dx$$

展開せずに解いてみる.

$x^3+2=t$ とおく

$3x^2\,dx=dt$

$$(与式)=\int t\,dt$$

$$=\frac{1}{2}t^2+C$$

$$=\frac{1}{2}(x^3+2)+C$$

例題2

$$\int \sin^2 x\cos x\,dx$$

$\sin x=t$ とおくと.

$\cos x\,dx=dt$

$$(与式)=\int t^2\,dt$$

$$=\frac{1}{3}t^3+C$$

$$=\frac{1}{3}\sin^3 x+C$$

例題2　続き

$$\int \frac{dx}{x\log x}$$

$\log x=t$ とおくと.

$$\frac{1}{x}\,dx=dt$$

$$(与式)=\int \frac{1}{t}\,dt$$

$$=\log|t|+C$$

$$=\log|\log x|+C$$

例題2 もういっちょ

$$\int \tan x \, dx$$

$$= \int \frac{\sin x}{\cos x} \, dx$$

$\cos x = t$ とおくと

$$-\sin x \, dx = dt$$

$$\therefore \sin x \, dx = -dt$$

$$(与式) = \int -\frac{1}{t} \, dt$$

$$= -\log|t| + C$$

$$= -\log|\cos x| + C$$

例題2 やれるもんなら

$$\int \frac{dx}{\cos^4 x}$$

$$\int \frac{1}{\cos^2 x}(\tan^2 x + 1) \, dx$$

$\tan x = t$ とおく

$$\frac{1}{\cos^2 x} \, dx = dt$$

$$(与式) = \int (t^2 + 1) \, dt$$

$$= \frac{1}{3}t^3 + t + C$$

$$= \frac{1}{3}\tan^3 x + \tan x + C$$