応用1

$$\boxed{\int \frac{dx}{\sin x}}$$

$$= \int \frac{\sin x}{\sin^2 x}\,dx$$

$$= \int \frac{\sin x}{(1-\cos^2 x)}\,dx$$

$\cos x = t$ とおくと

$-\sin x\,dx = dt$

$$(与式) = \int \frac{-1}{1-t^2}\,dt$$

$$= \int \frac{1}{t^2-1}\,dt$$

$$= \int \frac{1}{(t-1)(t+1)}\,dt$$

$$= \frac{1}{2}\int\left(\frac{1}{t-1} - \frac{1}{t+1}\right)dt$$

$$= \frac{1}{2}\left(\log|t-1| - \log|t+1|\right) + C$$

$$= \frac{1}{2}\log\left|\frac{\cos x - 1}{\cos x + 1}\right| + C$$

$$= \frac{1}{2}\log\frac{1-\cos x}{1+\cos x} + C$$

応用1 続き

$$\boxed{\int e^x \sin x\,dx} = -e^x\cos x + \int e^x \cos x\,dx$$

$$= -e^x\cos x + e^x \sin x - \int e^x \sin x\,dx$$

部分積分

$$\begin{pmatrix} f = e^x & f' = e^x \\ g' = \sin x & g = -\cos x \end{pmatrix}$$

$$\begin{pmatrix} f = e^x & f' = e^x \\ g' = \cos x & g = \sin x \end{pmatrix}$$

$$2\int e^x \sin x\,dx = e^x(\sin x - \cos x) + C$$

$$\int e^x \sin x\,dx = \frac{1}{2}e^x(\sin x - \cos x) + C$$

応用1 やれるもんなら.

$$I_n=\int(\log x)^n dx \quad (n \mid n \geqq 0 \text{の整数})$$

(1) $I_n = x(\log x)^n - nI_{n-1} \ (n \geqq 1)$

を証明.

(2) (1)を利用して I_2, I_3 を求めよ.

(1)

$$I_n=\int(\log x)^n dx$$

$$\left(\begin{array}{ll} f=(\log x)^n & f'=n(\log x)^{n-1}\cdot\dfrac{1}{x} \\ g'=1 & g=x \end{array}\right)$$

$$I_n=x(\log x)^n-\int n(\log x)^{n-1}dx$$

$$I_n=x(\log x)^n-nI_{n-1}$$

$$I_n=\int(\log x)^n dx \quad (n \mid n \geqq 0 \text{の整数})$$

(1) $I_n = x(\log x)^n - nI_{n-1} \ (n \geqq 1)$

を証明.

(2) (1)を利用して I_2, I_3 を求めよ.

(2) (1)より

$$I_2 = x(\log x)^2 - 2I_1$$

$$= x(\log x)^2 - 2\int \log x \, dx$$

$$= \underline{x(\log x)^2 - 2x\log x + 2x + C}$$

$$I_3 = x(\log x)^3 - 3I_2$$

$$= \underline{x(\log x)^3 - 3x(\log x)^2 + 6x\log x - 6x + C}$$